THE INSTRUMENTAL SYSTEM OF MECHANICS PROBLEMS ANALYSIS OF THE DEFORMED SOLID BODY

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In the abstract proposed is the Instrumental System of mechanics problems analysis of the deformed solid body. It supplies the researcher with the possibility to describe the input data on the object under analyses and the problem scheme based upon the variational principles within one task. The particular feature of System is possibility to describe the information concerning the object of any geometrical shape and the computation sheme according to the program defined for purpose. The Methods allow to compute the tasks with indefinite functional and indefinite geometry of the object (or the set of objects).

The System provides the possibility to compute the tasks with indefinite sheme based upon the Finite Element Method (FEM). The restrictions of the System usage are therefore determined by the restrictions of the FEM itself.

It contrast to other known programms using FEM (ANSYS, LS-DYNA and etc) described system possesses more universality in defining input data and choosing computational scheme.

Builtin is an original Subsytem of Numerical Result Analuses. It possesses the possibility to visualise all numerical results, build the epures of the unknown variables, etc. The Subsystem is approved while solving two- and three-dimensional problems of Elasticiti and Plasticity, under the conditions of Geometrical Unlinearity. Discused are Contact Problems of Statics and Dynamics.

1. Introduction

In the present a large number of programme systems, which allow to make numerical analysis of the strained-deformed solid body condition, has been worked out. These are such famous programmes as COSAR, ANSYS, LS-DYNA, PRO-ENGINEER, COMET/Acustics, COSMOS, etc. All of them make possible the solution of different types of applied and scientific problems. They possess the qualities that differ them from others such as high accuracy of the received results, convenience, orientation on the broad types of the problems, etc. But every concrete programme system is imperfect in solving the concrete problem, if the method of its solution was not provided by the designers. Besides if a user for solving the concrete problem is going to apply the method, which the system does not support. Other drawbacks of the systems are difficulties which may be caused by the user's insufficient knowledge of the programme systems and relatively high prices.

To overcome these drawbacks another approach to construct the systems is suggested. It is necessary to make the system which by the minimum incoming information given by the user about the investigated object and with the help of the large-block problem solution scheme could make the numerical calculation. To put it other it is necessary to make such a system which will enable the user to work not in the problem type but in the type of schemes and methods of their solution [1-5].

The article suggests the instrumental system which allows the user to make numerical modelling and the analysis of different mechanics problems of the deformed solid body by programming on his own the calculation scheme for the given problem on the special problem-oriented language in terms of variation calculus, and the laws and hypothesis of elasticity and plasticityon.

The instrumental system FORTU provides conducting of the analysis besides receiving of the numerical results.

The suggested system works absolutely in the interactive regime, that allows to make necessary corrections on all the stages of concrete problem solution.

2. The Instrumental System Structure

The system FORTU makes calculations on the basis of the Finite Element Method (FEM), which stipulates its structure. Functionally three blocks could be defined:

- The block of the initial data preparation consists of discretisation subsystem of the investigated object for the Finite Elements (FE), and the programming subsystem of the solution scheme description in FORTU language. This subsystem allows both to put the main calculating equations for the investigated object and to put the boundary (initial) problem conditions.
- The block of the interpretation of the programme given by the user makes the syntax and semantic control over the given incoming information, and in case of absence of the mistakes makes the calculations on the given programme.
- The block of the analysis of the received results makes it possible to visualise all the finite and some interim results of the calculation by the user's choice.

We have to admit that if the user is satisfied with the calculation results, he can get use of the built-in the system compiler from the RORTU language, which will allow him to get the fast programme, which implements the calculations according to the calculation programme set by him. For this programme the variations are admittable only for the main problem constants and for discretisation of the investigated object in FE.

3. The Language of Calculation Scheme Programming

The main calculus equations in the problems of the elasticity theory could be drawn and got directly from the Lagrange variation principle.

$$\delta(\Pi - A) = 0, \tag{3.1}$$

or from the Hamilton-Ostrogradsky principle

$$\delta\int_{\tau}(T - \Pi + A)d\tau = 0, \tag{3.2}$$

where T is a kinematic energy, Ï is a potential energy, A is the work of the inner and external forces [5]. To get the matrix equations of the concrete applied problem of the elasticity theory it is necessary to substitute the corresponding laws and hypothesis of the elasticity theory into the expressions (3.1) or (3.2). The suggested FORTU language makes it possible to describe these interrelations in the form similar to the natural one, while all the operations, necessary for getting the calculus equations (integration, differentiation, variation) will be made automatically according to the variation calculation and formal algebraic rules.After the main correlations are drawn the fivennn initial data will be substituted there and the calculations made.

Let us consider the structure and some basic operators of FORTU language using as example the solution of Hertz contact problem.

4. The Example of the FORTU Instrumental system application for the solution of Contact Problem.

Suppose, it is required to determine contact pressure in interaction of infinite elastic cylinder with elastic semiplate, Fig.1. Given contact interaction can be observed as the flat deformed condition.That is why contact interaction for the disk of R radius and the plate of 4R lenght along the axis Ox and 2R lenght along the axis Oy will be considered in this problem. The calculating scheme for the cylinder and for the plate was received with the aid of discretisation subsystem of the investigated object for the Finite Elements. For this problem linear triangle simplexes were used.

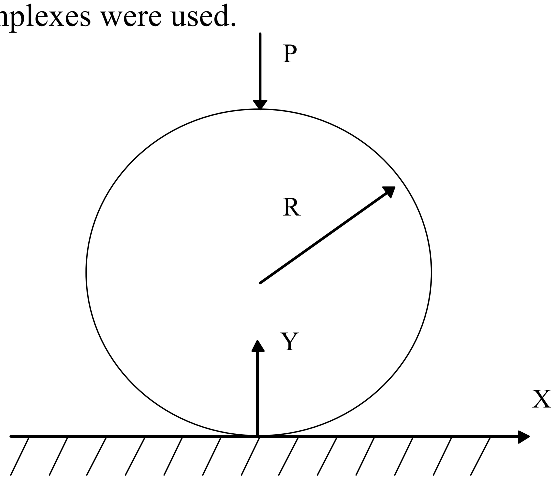

Fig 1. Contact interaction of the cylinder and semiplate

As the supposing zone width of contact doesn`t exceed 0.1R then vertical transferences are prohibited for plate junctions with the co-ordinates of y=-2R, and the horisontal transferences are prohibited for the junctions with the co-ordinates of x=2R and x=-2R. Such plate fixing almost doesn`tY influence on pressure - deformed condition of the interacting states.

Corresponding FORTU program is as follows:

```
OBJECT t(s2+b2)

s2.BEGIN
        RESULT u,v
        ARGUMENT x,y
        CONSTANT E,G,m,K,R
        FUNCTION Exx,Eyy,Gxy,Txy,Sxx,Syy
        RIGHT X,Y
        FUNCTIONAL U

        E       = 7*10^8
        m       = 0.3
        R         = 1
        K       = E / (1.0 - m^2)
        G       = E / (2.0 + 2.0 * m)
        Exx     = diff(u,x)
```

```
        Eyy       = diff(v,y)
        Gxy       = diff(u,y) + diff(v,x)
        Sxx        = K * (Exx + m* Eyy)
        Syy         = K * (Eyy + m* Exx)
        Txy        = G * Gxy
        U         = 0.5*Integral(Sxx & Exx + Syy & Eyy + Txy & Gxy)

        u(x=0)  = 0
        Y(0,2*R) = -1.96*10^6
END

b2.BEGIN
        RESULT u,v
        ARGUMENT x,y
         CONSTANT E,G,m,K,R
         FUNCTION Exx,Eyy,Gxy,Txy,Sxx,Syy
         RIGHT X,Y
         FUNCTIONAL U

        E         = 7*10^8
        m         = 0.3
        R         = 1
        K         = E / (1.0 - m^2)
        G         = E / (2.0 + 2.0 * m)
        Exx       = diff(u,x)
        Eyy     = diff(v,y)
        Gxy     = diff(u,y) + diff(v,x)
        Sxx     = K * (Exx + m* Eyy)
        Syy     = K * (Eyy + m* Exx)
        Txy     = G * Gxy

        U         = 0.5*Integral(Sxx & Exx + Syy & Eyy + Txy & Gxy)

        v(y=-2*R)=0
        u(x=2*R) =0
        u(x=0)=0
END
```

The solution results of the problem were compared with known analitical solution. If $E^{(1)} = E^{(2)}, \nu^{(1)} = \nu^{(2)} = 0.3$, then maximum contact pressure in the centre of the contact ground is described by the formule $q_0 = 0.418\sqrt{pE/R}$ and ground width of the contact $2S = 1.52\sqrt{\frac{P}{ER}}$ [7].

In the Figure 2 the fraph of the contact pressure distribution is given in relative co-ordinates (dotted line - the solution received with the aid of FORTU, constant line - the solution calculated according to Hertz formule). While breaking the cylinder and the plate into 144 FE the biggest deviation from the exact solution does not exceed 3%. While condensing FE - net in two time the error was 0.8%. Received solution was compared with other published solutions of this problem [8,9].

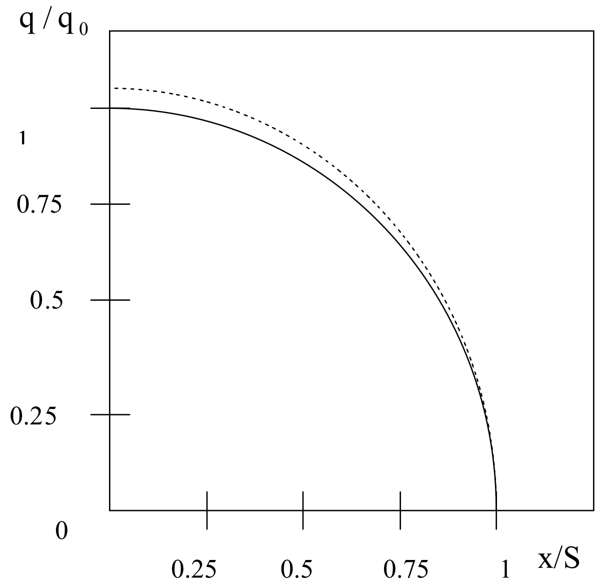

Fig 2. Contact Pressure Distribution

The analisis subsystem allows any quantity visualization formed by user in FORTU terms. For example,the tangent stresses distribution on the cylinder and the plate is given in Fig.3 (in one and the same colour scale).

The given problem was solved in the computer with the processor Pentium of the tact frequency 75 MHz and memory 16 Mb. The calculating time was 2 minutes.

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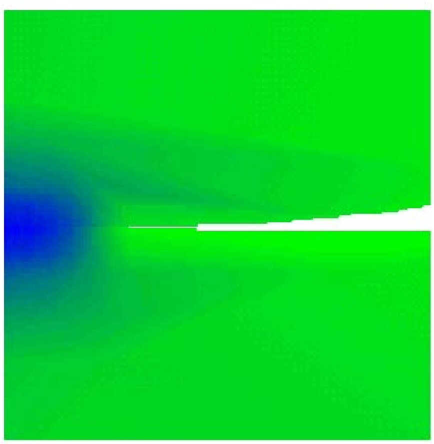

Fig3. Syy Function in contact zone

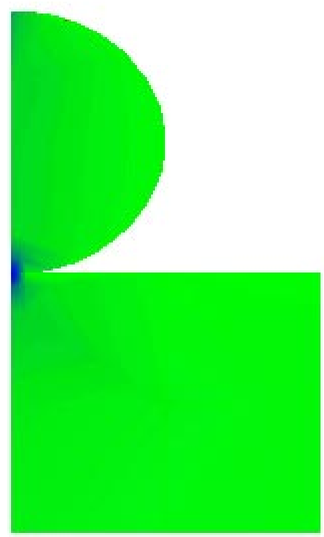

Fig 4. Syy Function for cylinder and semiplate

5. Literature

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